

Math AS1: Limit Exercises

Find each of the following limits, giving the left and right limits whenever appropriate.

1. $\lim_{x \rightarrow 2} (3x^2 - 4x + 1)$
2. $\lim_{x \rightarrow 2} \frac{x - 2}{4 - x^2}$
3. $\lim_{x \rightarrow 4} \frac{x - 4}{x^2 - x - 12}$
4. $\lim_{x \rightarrow 1} \frac{x - 4}{x^2 - x - 12}$
5. $\lim_{x \rightarrow -3} \frac{x - 4}{x^2 - x - 12}$
6. $\lim_{x \rightarrow -4} \frac{2x^2 + 5x - 12}{x^2 + 4x}$
7. $\lim_{x \rightarrow 2} \frac{3x^2 - 7x + 2}{2x^2 - 3x - 2}$
8. $\lim_{x \rightarrow \frac{1}{2}} \frac{2x - 1}{8x^3 - 1}$
9. $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 7x + 10}$
10. $\lim_{x \rightarrow -2} \frac{2x^2 + 5x - 6}{x^2 + 4}$
11. $\lim_{x \rightarrow 1/2} \frac{2x^2 + 5x - 3}{4x^2 - 8x + 3}$
12. $\lim_{x \rightarrow 1} \frac{x^3 + 2x^2 - 4x + 1}{x^2 + 3x - 4}$
13. $\lim_{x \rightarrow 8} \frac{x^2 + 4x - 5}{x^2 - 9x + 8}$
14. $\lim_{x \rightarrow 1} \frac{x^2 + 4x - 5}{x^2 - 9x + 8}$
15. $\lim_{x \rightarrow 0} \frac{x^2 + 4x - 5}{x^2 - 9x + 8}$
16. $\lim_{x \rightarrow -3} \frac{x^3 + 27}{x^2 + 3x}$
17. $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{2x^2 - 2x + 3}$
18. $\lim_{h \rightarrow 0} \frac{(x + h)^2 - x^2}{h}$
19. $\lim_{h \rightarrow 0} \frac{(x + h)^3 - x^3}{h}$
20. $\lim_{x \rightarrow 2} \frac{\sqrt{5 - x} - \sqrt{3}}{2 - x}$
21. $\lim_{x \rightarrow 2} \frac{\sqrt{2x + 5} - 3}{x - 2}$
22. $\lim_{x \rightarrow 10} \frac{\sqrt{2x + 5} - 3}{x - 2}$
23. $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 7} - 4}{x - 3}$
24. $\lim_{x \rightarrow 4} \frac{\sqrt{x + 5} - 3}{x^2 - 16}$
25. $\lim_{x \rightarrow 3} \frac{2 - \sqrt{x + 1}}{9 - x^2}$
26. $\lim_{x \rightarrow 4} \frac{\frac{1}{4} - \frac{1}{x}}{x - 4}$
27. $\lim_{x \rightarrow 2} \frac{\frac{x}{2x - 1} - \frac{2}{3}}{x - 2}$
28. $\lim_{x \rightarrow -3} \frac{\frac{1}{x} + \frac{1}{3}}{x^2 - 9}$
29. $\lim_{h \rightarrow 0} \frac{\frac{1}{x + h} - \frac{1}{x}}{h}$
30. $\lim_{x \rightarrow -2} \left(\frac{4}{x + 2} + \frac{8}{x^2 + 2x} \right)$
31. $\lim_{x \rightarrow 2} \left(\frac{1}{2x^2 - 3x - 2} - \frac{1}{x^2 + x - 6} \right)$
32. $\lim_{x \rightarrow -2} \begin{cases} x^2 - 3, & \text{if } x \leq -2 \\ x + 4, & \text{if } x > -2 \end{cases}$
33. $\lim_{x \rightarrow 2^-} \begin{cases} x^2 + 5, & \text{if } x < 2 \\ x + 4, & \text{if } x > 2 \end{cases}$
34. $\lim_{x \rightarrow -3} \frac{|2x + 6|}{x + 3}$
35. $\lim_{x \rightarrow 0} \frac{3x + |x|}{x}$
36. $\lim_{x \rightarrow 0} \frac{\sin(-3x)}{2x}$
37. $\lim_{x \rightarrow 0} \frac{\tan 3x}{\tan 5x}$
38. $\lim_{x \rightarrow 5} \sqrt{x^2 + 2x - 11}$
39. $\lim_{x \rightarrow 7} \sqrt{14 - 2x}$
40. $\lim_{x \rightarrow 3} \sqrt{x^2 - 7x + 12}$
41. $\lim_{x \rightarrow -1} \frac{x - 2}{x + 1}$
42. $\lim_{x \rightarrow 1} \frac{1}{x^2 - 2x + 1}$
43. $\lim_{x \rightarrow -2} \frac{x + 3}{2x^2 + 3x - 2}$
44. $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 8x + 16}$
45. $\lim_{x \rightarrow 1} \frac{x^2 - x - 6}{x^3 - 5x^2 + 7x - 3}$
46. $\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x^3 + 3x^2 - 4}$
47. $\lim_{x \rightarrow 13} \sqrt{\frac{x + 4}{x - 13}}$
48. $\lim_{x \rightarrow 0} \frac{1}{\sqrt{x^2 - 5x}}$
49. $\lim_{x \rightarrow \infty} \frac{(2x + 3)(4 - x)}{x^2 - 1}$
50. $\lim_{x \rightarrow \infty} \frac{x^3 - 3x + 7}{3 - x^2}$
51. $\lim_{x \rightarrow -\infty} \frac{4x^6 - 10x^4 + 2x^2}{10x^6 - 8x^5 + 11x^2}$
52. $\lim_{x \rightarrow \infty} \frac{2x^3 - 4x^2 + 9}{3x - 12x^3 + 15x^2}$
53. $\lim_{x \rightarrow -\infty} \frac{5 - 6x}{\sqrt{4x^2 - x + 3}}$
54. $\lim_{x \rightarrow -\infty} \frac{3x^2 + 6x}{\sqrt{2x^2 - x}}$
55. $\lim_{x \rightarrow \infty} \frac{3x - 2}{\sqrt{2x - x^3}}$
56. $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x - 1} - \sqrt{x^2 + 2x + 1})$

Answers

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|--|------------------|---|---|
| 1. 5 | 16. -9 | 32. $\lim_{x \rightarrow -2^-} = 1$
$\lim_{x \rightarrow -2^+} = 2$ | 44. $\lim_{x \rightarrow 4^-} = -\infty$
$\lim_{x \rightarrow 4^+} = \infty$ |
| 2. $-1/4$ | 17. 0 | 33. 9 | 45. ∞ |
| 3. $1/7$ | 18. $2x$ | 34. $\lim_{x \rightarrow -3^-} = -2$
$\lim_{x \rightarrow -3^+} = 2$ | 46. $\lim_{x \rightarrow -2^-} = \infty$
$\lim_{x \rightarrow -2^+} = -\infty$ |
| 4. $1/4$ | 19. $3x^2$ | 35. $\lim_{x \rightarrow 0^-} = 2$
$\lim_{x \rightarrow 0^+} = 4$ | 47. $\lim_{x \rightarrow 13^-}$ dne
$\lim_{x \rightarrow 13^+} = \infty$ |
| 5. $\lim_{x \rightarrow -3^-} = -\infty$
$\lim_{x \rightarrow -3^+} = \infty$ | 20. $\sqrt{3}/6$ | 36. $-3/2$ | 48. $\lim_{x \rightarrow 0^-} = \infty$
$\lim_{x \rightarrow 0^+}$ dne |
| 6. $11/4$ | 21. $1/3$ | 37. $3/5$ | 49. -2 |
| 7. 1 | 22. $1/4$ | 38. $2\sqrt{6}$ | 50. $-\infty$ |
| 8. $1/3$ | 23. $3/4$ | 39. $\lim_{x \rightarrow 7^-} = 0$
$\lim_{x \rightarrow 7^+}$ dne | 51. $2/5$ |
| 9. -4 | 24. $1/48$ | 40. $\lim_{x \rightarrow 3^-} = 0$
$\lim_{x \rightarrow 3^+}$ dne | 52. $-1/6$ |
| 10. -1 | 25. $1/24$ | 41. $\lim_{x \rightarrow -1^-} = \infty$
$\lim_{x \rightarrow -1^+} = -\infty$ | 53. 3 |
| 11. $-7/4$ | 26. $1/16$ | 42. ∞ | 54. ∞ |
| 12. $3/5$ | 27. $-1/9$ | 43. $\lim_{x \rightarrow -2^-} = \infty$
$\lim_{x \rightarrow -2^+} = -\infty$ | 55. dne |
| 13. $\lim_{x \rightarrow 8^-} = -\infty$
$\lim_{x \rightarrow 8^+} = \infty$ | 28. $1/54$ | | 56. $1/2$ |
| 14. $-6/7$ | 29. $-1/x^2$ | | |
| 15. $-5/8$ | 30. -2 | | |
| | 31. $-1/25$ | | |